Volatility Index

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1 Introduction

Efficiency/Volatility is a fundamental part of math, but there doesn't seem to be a defined equation and index for determining the volatility of a function. Defining such a function could have useful implications in analysis of functions, especially in machine learning.

2 Volatility in 2 Dimensions

The volatility of function $f(x)$ can be written in the following form:

$$V(a,\ b)\ =\ \frac{1}{b-a}\int_a^b \sqrt{1\ +\ \left(\frac{dy}{dx}\right)^2}dx$$

This equation divides the arc length of a function by the horizontal distance covered, providing a value in the bounds $[1, \infty)$. 1 represents a horizontal line, with higher values representing functions that fluctuate from the horizontal.

$$V_d(a,\ b)\ =\ \frac{1}{\sqrt{(b-a)^2+(f(b)-f(a))^2}}\int_a^b \sqrt{1\ +\ \left(\frac{dy}{dx}\right)^2}dx$$

This equation divides arc length by the length of the axis connecting the two points (a, b) rather than the horizontal axis.

3 Volatility in Higher Dimensions

In 3 dimensions, $z = f(x, y)$:

$$V = \iint_A \frac{\sqrt{[f_x]^2+[f_y]^2+1}}{Area\ of\ A}\,dA$$

In an n-dimensioned function, f:

$$V = \overbrace{\int \cdots \int_M}^{\text{n amount of integrals}} \frac{\sqrt{\nabla f \circ \nabla f + 1}}{Space\, Consumed\ by\ M}\, dM$$

Here, a gradient is used to represent all the partial derivatives, and it is multiplied by itself in a Hadamard Product to square each partial derivatives. For dimensions ≥ 3, I still don't know exactly what would be put in the denominator.

4 Conclusion

Having an equation for volatility, especially one in all dimensions could be useful to analyze and compare different functions. An end goal would be to put the feed-forward function of a neural network in the volatility equation, possibly giving more insight on the network.